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MODELING OF THE PROCESS OF DEFORMATION OF THE ELASTIC RACK OF THE WORKING BODIES OF THE TILLAGE IMPLEMENT

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Abstract. The article presents the results of theoretical studies of the dynamic model of the process of deformation of the elastic rack of a disk tool of arbitrary shape, a system of differential equations in general and developed the corresponding program code in Mathematic software package. Taking the form of an elastic discus disk for an Archimedean spiral, when the functions of its boundaries are given in polar coordinates $f_1(\theta)=\frac{a\theta}{2\pi}+b$, $f_2(\theta)=\frac{a\theta}{2\pi}+b+h$, $\theta_s \le \theta \le \theta_f$, where the parameters of the geometric shape a (spiral pitch), b (spiral displacement along the radial coordinate), h (elastic column thickness) are determined by its equivalent physical a mathematical model in the form of a rigid mathematical pendulum of length l, to the load of which are attached two springs along the axes Ox and Oz with stiffness coefficients k_x and k_z, respectively, which deflect it by an angle φ. The dependences of the stiffness coefficients k_x and k_z, the length l and the angle φ of the equivalent physico-mathematical model of the elastic stand of the disc with the parameters of the geometric shape a=0.8 m, b=0 m, h=0.01 m on the values of F_{ex} and F_{ez}, acting on the free end of the rack along the axes Ox and Oz.

Key words: disk, elastic disk rack, disk tool, disk, deformation of elastic rack.

FORMULATION OF THE PROBLEM

One of the main tasks of tillage is to create favorable conditions for the accumulation of nutrients and, especially, moisture for the normal development of crops. The key to the successful course of these processes, according to agronomists and soil scientists, is a homogeneous aggregate composition of the soil throughout the depth of cultivation. Accordingly, it is necessary to improve agricultural machinery and implements in order to ensure optimal modes of operation while reducing energy costs for the process.

Of particular importance is the solution of these problems for tillage equipment, including disk working bodies, as they provide 60 - 80% of the preliminary and main tillage while reducing up to 20% of energy consumption for the process.

A promising direction to improve the quality of tillage while reducing the energy consumption of the process is the use of disk tools with individual mounting of working bodies on elastic racks. This causes their oscillations due to the uneven forces of soil resistance and its destruction with lower energy consumption and better adaptation to the terrain, which increases the possibility of ensuring a given quality of cultivation.

In order to increase the efficiency of disk tools, it is important to establish patterns of influence of the parameters of the elastic rack of the disk tool on its geometric dimensions and values of external forces acting on the rack during the process.

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

The analysis of researches in the direction of increase of efficiency of tillage aggregates with disk working bodies specify expediency of application of their individual fastening on elastic racks (Haidenko *et. al* 2013; Hrynenko *et. al* 2011; Shevchenko 2016). In (Haponenko 2017) a mathematical model of the motion of an elastic riser with a spherical disk was developed, its design parameters and dynamic characteristics and their influence on the oscillations of the riser due to non-stationary process and changes in resistance during operation of the unit. The author found that the use of elastic risers with certain parameters, compared with a typical elastic riser, the parameters of which are justified only by the functional need to protect the working body from overload, reduces energy consumption of the tillage process by 7% in compliance with agronomic requirements.

Considering the characteristics of the regenerative force of an elastic strut of any configuration based on the theory of nonlinear problems of statics of thin rods in (Kushnarev *et. al* 2008; Kushnarov 1999) developed an analytical method for determining it, when the designed riser is divided into a number of segments, lines and arcs with a certain radius.

Kushnaryov developed an analytical method for determining the characteristics of the recovery force of a riser of any configuration based on the theory of nonlinear problems of statics of thin rods (Kushnarev *et. al* 2008; Kushnarov 1999). To analytically determine the regenerative force, the designed riser of any configuration is divided into a number of segments, lines and arcs with a certain radius.

The author (Donchenko 2004) investigated the nature of self-oscillations that occur in elastic racks in the process of interaction of the working body of the aggregate with the soil. The method of calculation of the main characteristics and parameters of the oscillatory system on the basis of the approximate method of harmonic balance known in nonlinear mechanics is developed in the work. According to the results of the study, numerical solutions of differential equations describing the mathematical model of the system are performed, when different phases of oscillations during the period are described by different differential equations.

In the work of I. A. Shevchenko (Shevchenko 1988) the non-autonomous problem of action of soil on the working body is considered, when in the first approximation the working body of the tool with S-shaped riser is presented as a material point with superimposed elastic-viscous connections superimposed on it, with directions of generalized coordinates. In studies (Shevchenko 1988) on the analysis of the spectral density of traction resistance, it was concluded that the random function of the resistance of the working body in the soil environment can be replaced by a harmonic function or the sum of two or three harmonic functions

$P(t) = q_1 \cos w_1 t + q_2 \cos w_2 t + q_3 \cos w_3 t$.

The stiffness of S-shaped risers is approximated by a cubic parabola of the form: $F = aq + bq^3$, where a is the stiffness coefficient that connects the linear deformation with the applied force; b – stiffness coefficient, which reflects the non-linearity of the relationship between the regenerative force and displacement.

Consideration of the known models of functioning of working bodies of tillage implements (Labatyuk *et. al* 2015), including on the elastic riser, indicates the expediency of taking into account the empirical characteristics and design parameters of the object of study with matrix coefficients, in particular in (Shevchenko 1988). The use of more complex models in research (Donchenko 2004; Shevchenko 1988) makes it almost impossible to analytically solve the problem. This determines the feasibility of further theoretical research in this area.

THE PURPOSE OF RESEARCH

The aim is a theoretical study of the process of deformation of the elastic rack of the disc cultivator of any shape under the action of external forces arising in the process of interaction of the working body of the aggregate with the tillage.

To achieve this goal it is necessary to solve the following tasks:

– to investigate the dynamic model of the process of deformation of the elastic rack of the disc cultivator of any shape under the action of external forces;

– to make a system of differential equations in general form and to develop the corresponding program code which allows to define stresses, relative and absolute deformations in each point of an elastic rack of the disc cultivator;

– to determine the dependences of the parameters of the equivalent physical and mathematical model of the elastic rack of the disc cultivator with its geometric dimensions and values of external forces acting on the free end of the rack.

RESEARCH RESULTS

The study of the process of deformation of the elastic rack of the disc cultivator will be considered taking into account the following assumptions and simplifications:

– the elastic rack is absolutely elastic, i.e. its state can be described by the equation of equilibrium, the equations of Hooke's law and the relationship between the components of the strain tensor and the components of the displacement vector;

– the deformation process occurs in two directions, so we will consider a flat coordinate system;

– the elastic rack has a spiral shape and can be described by a function in the polar coordinate system.

The calculated scheme of the process of deformation of the elastic strut is presented in Fig. 1. The center of coordinates is at point 0.

The function describing the boundaries of the elastic rack is written in the form:

– in the polar coordinate system (r, θ):

border A\`B\`: $r_1 = f_1(\theta_1)$, де $\theta_s \le \theta_1 \le \theta_f$, (1)

border AB: $r_2 = f_2(\theta_2)$, де $\theta_s \le \theta_2 \le \theta_f$, (2)

border AA\`: $\theta \approx \theta_s = const$, (3)

border BB\`: $\theta \approx \theta_f = const$, (4)

where r is the radial coordinate of the point in the polar coordinate system, m; θ – angular coordinate of a point in the polar coordinate system; indices "1" and "2" correspond to the inner and outer boundaries of the elastic strut; indices "s" and "f" correspond to the initial and final angles of the boundary of the elastic strut; whereas the distance between the boundaries A\`B\` and AB (thickness) of the elastic strut is much smaller than its other geometric dimensions, for the boundary AA\` and BB\` identical sign "≈";

– in the Cartesian coordinate system (x, z):

border A\`B\`:

$$\begin{cases} x_1 = r_1 \cos\theta_1 = f_1(\theta_1)\cos\theta_1, \\ z_1 = r_1 \sin\theta_1 = f_1(\theta_1)\sin\theta_1, \\ \theta_s \le \theta_1 \le \theta_f, \end{cases} \quad (5)$$

border AB:

$$\begin{cases} x_2 = r_2 \cos\theta_2 = f_2(\theta_2)\cos\theta_2, \\ z_2 = r_2 \sin\theta_2 = f_2(\theta_2)\sin\theta_2, \\ \theta_s \le \theta_2 \le \theta_f, \end{cases} \quad (6)$$

border AA\`: $z = const$, (7)

border BB\`: $x = const$, (8)

where x, z is the coordinate of the point in the Cartesian coordinate system, m.

In addition to the above description of the boundaries of the elastic strut, it can be represented as an equivalent physical-mathematical model: a rigid mathematical pendulum of length l, to the load of which are fixed two springs along the axes Ox and Oz with stiffness coefficients k_x and k_z, respectively, deflecting it by angle φ. That is, the load of the mathematical pendulum is subject to two additional elastic forces along the axis Ox and Oz: F_{ex} and F_{ez}, respectively, which can be represented by Hooke's law for the spring in the form (Labatyuk *et. al* 2015):

$$F_{ex} = k_x \Delta x_B, \quad (9)$$

$$F_{ez} = k_z \Delta z_B, \quad (10)$$

where Δx_B, Δz_B – absolute displacements of the point B (or B ') in the Cartesian coordinate system as a result of deformation of the elastic strut, m.

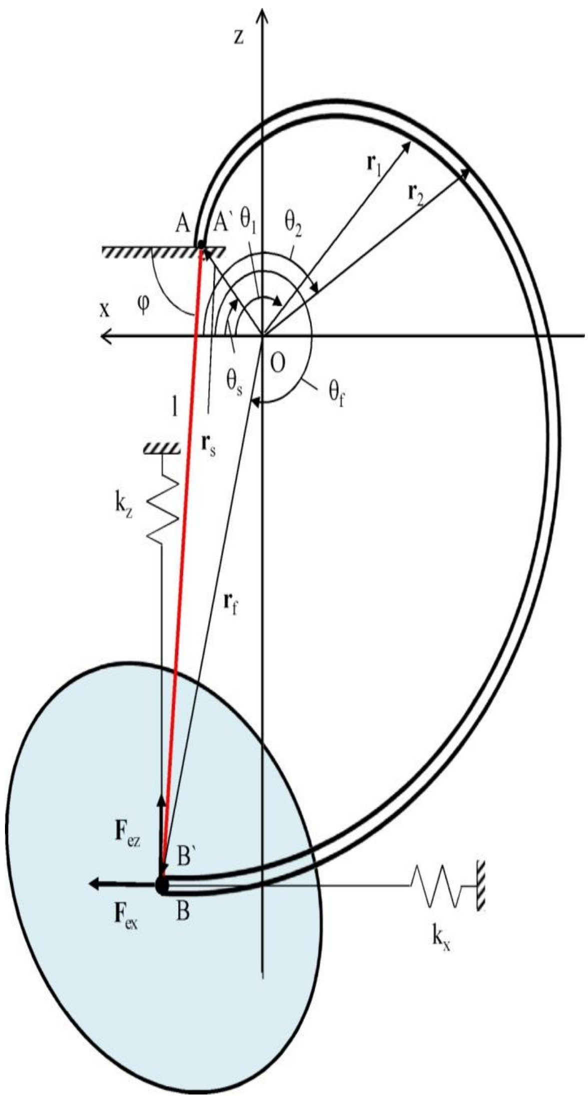

Figure 1. Calculation scheme of the process of deformation of the elastic strut.

To move from a real to an equivalent physical-mathematical model, it is necessary to determine the dependences of the absolute displacements Δx_B, Δz_B and the stiffness coefficients k_x and k_z on the geometrical parameters of the elastic rack and the elastic properties of the material. And also establish the dependence of the length l on the angle φ.

Accepting mass forces equal to zero, the equation of equilibrium of the points of the elastic post in the polar coordinates has the form (Bezukhov 1953):

$$\begin{cases} \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0, \\ \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + 2\frac{\sigma_{rr}}{r} = 0, \end{cases} \quad (11)$$

where σ_{rr}, $\sigma_{\theta\theta}$ – normal voltage, Pa; $\sigma_{r\theta}$ – tangential voltage, Pa. The relationships between the components of the strain tensor and the components of the displacement vector in polar coordinates can be represented as (Demydov 1979):

$$\begin{cases} \varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \\ \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}, \\ \varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right), \end{cases} \quad (12)$$

where $\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{r\theta}$, – relative deformations; u_r, u_θ – absolute deformations.

The equation of Hooke's law in polar coordinates has the form (Sausvell 1948):

$$\begin{cases} \varepsilon_{rr} = \frac{1}{E}\left(\left(1-\nu^2\right)\sigma_{rr} - (1+\nu)\nu\sigma_{\theta\theta}\right), \\ \varepsilon_{\theta\theta} = \frac{1}{E}\left(\left(1-\nu^2\right)\sigma_{\theta\theta} - (1+\nu)\nu\sigma_{rr}\right), \\ \varepsilon_{r\theta} = \frac{1}{E}(1+\nu)\sigma_{r\theta}, \end{cases} \quad (13)$$

where E is the Young's modulus of elasticity of the material of the elastic rack, kPa; ν is the Poisson's ratio of the elastic strut material.

To solve equations (11) - (13) together, we introduce the Erie function in the polar coordinates Φ (r, θ). The voltage can be determined as follows (Tymoshenko *et. al* 1973):

$$\begin{cases} \sigma_{rr} = \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2}, \\ \sigma_{\theta\theta} = \frac{\partial^2 \Phi}{\partial r^2}, \\ \sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Phi}{\partial \theta}\right). \end{cases} \quad (14)$$

For the functions σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ to correspond to expressions (14), the Erie function in the polar coordinates Φ (r, θ) must obey the biharmonic equation (Shemiakyn 1968):

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2}\right) = 0. \quad (15)$$

According to research (Byrher *et. al* 1986): for a circular ring or part of it, the voltages are proportional to cosθ or sinθ, i.e. the Erie function can be represented as:

$$\begin{aligned} \Phi(r,\theta) = {} & \left(R_1(\theta)\cos\theta + B_1 r\theta\sin\theta\right) + \\ & + \left(R_2(\theta)\sin\theta + B_2 r\theta\cos\theta\right) \end{aligned} \quad (16)$$

where $R_1(\theta)$, $R_2(\theta)$ are free functions; B_1, B_2 – free constants.

Substituting (16) into (15) and solving the obtained equation in the software package Mathematica we obtain the values of free functions:

$$\begin{aligned} R_1(\theta) &= \frac{A_{11}}{r} + A_{21}r + A_{31}r^3 + A_{41}r\ln(r), \\ R_2(\theta) &= \frac{A_{12}}{r} + A_{22}r + A_{32}r^3 + A_{42}r\ln(r), \end{aligned} \quad (17)$$

where A_{11}, A_{21}, A_{31}, A_{41}, A_{12}, A_{22}, A_{32}, A_{42} – free constants.

Substituting (17) into (16) we obtain the Erie function for an elastic rack:

$$\Phi(r,\theta)=\begin{pmatrix}\frac{A_{11}}{r}+A_{21}r+A_{31}r^3+\\+A_{41}r\ln(r)\end{pmatrix}\cos\theta+$$

$$+B_1 r\theta\sin\theta+ \qquad (18)$$

$$+\begin{pmatrix}\frac{A_{12}}{r}+A_{22}r+A_{32}r^3+\\+A_{42}r\ln(r)\end{pmatrix}\sin\theta+B_2 r\theta\cos\theta.$$

From the system of equations (14) and the obtained Erie function (18) we find the functions of all voltages arising in the elastic rack:

$$\begin{cases}\sigma_{rr}=\frac{1}{r^3}\left(-2A_{11}\cos\theta-2A_{12}\sin\theta+\ldots\right.\\ +A_{41}\cos\theta+A_{42}\sin\theta+2B_1\cos\theta-\ldots\\ \sigma_{\theta\theta}=2B_1 r\cos\theta-B_1 r\theta\sin\theta-B_2 r\theta\ldots\\ \times\left(\frac{A_{11}}{r}+r\left(A_{21}+A_{31}r^2+A_{41}\ln r\right)\right)-\ldots\\ \sigma_{r\theta}=\frac{1}{r^3}\left(-2A_{11}\sin\theta-2A_{12}\cos\theta+\right.\\ \left.+A_{41}\sin\theta-A_{42}\cos\theta\right)\Big).\end{cases} \qquad (19)$$

The next step is to determine the constants A_{11}, A_{21}, A_{31}, A_{41}, A_{12}, A_{22}, A_{32}, A_{42}, B_1, B_2 taking into account the following boundary conditions of the elastic strut:

$$\begin{cases}\sigma_{rr}\big|_{r=f_1(\theta)}=\sigma_{rr}\big|_{r=f_2(\theta)}=0,\\ \sigma_{r\theta}\big|_{r=f_1(\theta)}=\sigma_{r\theta}\big|_{r=f_2(\theta)}=0,\\ \int_{f_1(\theta)}^{f_2(\theta)}\sigma_{r\theta}\big|_{\theta=\theta_s}=0,\\ \int_{f_1(\theta)}^{f_2(\theta)}\sigma_{\theta\theta}\big|_{\theta=\theta_s}=0,\\ \int_{f_1(\theta)}^{f_2(\theta)}\sigma_{r\theta}\big|_{\theta=\theta_f}=F_{ex},\\ \int_{f_1(\theta)}^{f_2(\theta)}\sigma_{\theta\theta}\big|_{\theta=\theta_f}=F_{ez}.\end{cases} \qquad (20)$$

Substituting (19) into the equations of Hooke's law (13) and dependences (12) we obtain a system of differential equations with respect to the absolute deformations $u_r(r, \theta)$ and $u_\theta(r, \theta)$:

$$\begin{cases}\frac{\partial u_r}{\partial r}=\frac{1}{E}\left(\left(1-\nu^2\right)\frac{1}{r^3}\left(-2A_{11}\cos\theta-2A\ldots\right.\right.\\ +A_{41}\cos\theta+A_{42}\sin\theta+2B_1\cos\theta-2\ldots\\ -B_2 r\theta\cos\theta-2B_2 r\sin\theta-\cos\theta\left(\frac{A_{11}}{r}\ldots\right.\\ -\sin\theta\left(\frac{A_{12}}{r}+r\left(A_{22}+A_{32}r^2+A_{42}\ln r\right)\ldots\right.\\ \frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{r}=\frac{1}{E}\left(\left(1-\nu^2\right)\left(2B_1 r\cos\theta-\ldots\right.\right.\\ -\cos\theta\left(\frac{A_{11}}{r}+r\left(A_{21}+A_{31}r^2+A_{41}\ln r\right)\ldots\right.\\ -(1+\nu)\nu\frac{1}{r^3}\left(-2A_{11}\cos\theta-2A_{12}\sin\theta\ldots\right.\\ +A_{41}\cos\theta+A_{42}\sin\theta+2B_1\cos\theta-2\ldots\\ \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta}+\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}\right)=\frac{1}{E}(1+\nu)\frac{1}{r^3}\left(-\ldots\right.\\ +r^2\left(2A_{31}r^2\sin\theta-2A_{32}r^2\cos\theta++A\ldots\right.\end{cases} \qquad (21)$$

Solving the resulting system of equations in the software package Mathematica we obtain the functions of absolute deformations $u_r(r, \theta)$ and $u_\theta(r, \theta)$:

$$\left\{\begin{array}{l} u_r(r,\theta)=\dfrac{1+\nu}{4E}\left(-A_{11}\dfrac{4(\nu-1)}{r^2}+A_{31}\nu r^4+2A_{41}\nu r^2\ln r+4\ln r\times\right. \\ \left.\times\left(\nu A_{11}-(\nu-1)(A_{41}+2B_1)\right)+r^2\left(2\nu A_{21}-4A_{31}(\nu-1)-\nu(A_{41}+4B_1)\right)\right), \\ u_\theta(r,\theta)=\dfrac{1+\nu}{Er^2}\left(\theta\left(-\left(-2+\left(2+r^2\right)n\right)A_{11}+r^2\left(r^2\left(-\nu A_{21}-\left(2+\left(r^2-2\right)\nu\right)A_{31}\right)+\right.\right.\right. \\ \left.\left.+\left(\nu-1-r^2\nu\ln r\right)A_{41}+2\left(\nu-1+r^2\nu\right)B_1\right)\right)+\cos\theta\left(4(\nu-1)A_{12}-2r^4(\nu-1)A_{22}+\right. \\ \left.+4r^4A_{32}+4r^6A_{32}+4r^4\nu A_{32}-4r^6\nu A_{32}+2r^2A_{42}+r^4A_{42}-r^4\nu A_4\right)^2+ \\ \left.+2r^4\ln r\,A_{42}-2r^4\nu\ln r\,A_{42}+2r^4\theta B_1-2r^4\theta\nu B_1+2r^4B_2-2r^4\nu B_2\right)+ \\ +\sin\theta\left(-4(\nu-1)A_{11}+r^2\left(-2A_{41}+r^2\left(2(\nu-1)A_{21}+4\left(-1+r^2(\nu-1)-\nu\right)A_{31}+\right.\right.\right. \\ \left.\left.\left.\left.+(\nu-1)\left((1+2\ln r)A_{41}-2B_1+2\theta B_2\right)\right)\right)\right)\right). \end{array}\right. \qquad (22)$$

Taking into account the obtained absolute deformations (21) we obtain new coordinates of the position of the points B after the deformation of the elastic strut

$$\begin{cases} x_B=\left(f_2(\theta_f)+u_r\left(f_2(\theta_f),\theta_f\right)\right)\cos\theta_f, \\ z_B=\left(f_2(\theta_f)+u_r\left(f_2(\theta_f),\theta_f\right)\right)\sin\theta_f. \end{cases} \qquad (23)$$

Then the length l_k and the angle φ according to fig. 1 is presented as an expression:

$$l_k=\sqrt{(x_B-x_A)^2+(z_B-z_A)^2}=$$
$$=\left(\left(\frac{\left(f_2(\theta_f)+u_r\left(f_2(\theta_f),\theta_f\right)\right)}{\cos\theta_f-f_2(\theta_s)\cos\theta_s}\right)^2+\right. \qquad (24)$$
$$\left.+\left(\frac{\left(f_2(\theta_f)+u_r\left(f_2(\theta_f),\theta_f\right)\right)}{\sin\theta_f-f_2(\theta_s)\sin\theta_s}\right)^2\right)^{1/2},$$

$$\varphi=arctg\frac{z_B-z_A}{x_B-x_A}=$$
$$=arctg\frac{\dfrac{\left(f_2(\theta_f)+u_r(f_2(\theta_f),\theta_f)\right)}{\sin\theta_f-f_2(\theta_s)\sin\theta_s}}{\dfrac{\left(f_2(\theta_f)+u_r(f_2(\theta_f),\theta_f)\right)}{\cos\theta_f-f_2(\theta_s)\cos\theta_s}}. \qquad (25)$$

The absolute displacements of point B in the Cartesian coordinate system as a result of deformation of the elastic strut are as follows:

$$\Delta x_B=u_r\left(f_2(\theta_f),\theta_f\right)\cos\theta_f, \qquad (26)$$

$$\Delta z_B=u_r\left(f_2(\theta_f),\theta_f\right)\sin\theta_f. \qquad (27)$$

Considering (9) - (10), (26) - (27) we obtain the stiffness coefficients k_x and k_z

$$k_x=\frac{F_{ex}}{u_r\left(f_2(\theta_f),\theta_f\right)\cos\theta_f}, \qquad (28)$$

$$k_z=\frac{F_{ez}}{u_r\left(f_2(\theta_f),\theta_f\right)\sin\theta_f}. \qquad (29)$$

The joint solution of the obtained systems of equations (19)-(29) in general by analytical methods is a very complex process, so we turn to the solution of a specific problem.

The analysis of elastic bars for disc cultivator allows for the first approximate their shape approximated function Archimedean spiral, i.e.

$$f_1(\theta)=\frac{a\theta}{2\pi}+b\text{, де } \theta_s \le \theta \le \theta_f, \qquad (30)$$

$$f_2(\theta)=f_1(\theta)+h=\frac{a\theta}{2\pi}+b+h\text{, де } \theta_s \le \theta \le \theta_f, \qquad (31)$$

where *a* is the step of the spiral, m; *b* is the displacement of the spiral along the radial coordinate, m; *h* is the thickness of the elastic rack, m.

To determine the initial θ_s and final θ_f angles, we use the conditions of the horizontal and vertical tangent to the function of the Archimedes spiral:

$$\frac{dx}{dz}(\theta_s)=0\text{, де } 0 \le \theta_s \le \frac{\pi}{2}, \qquad (32)$$

$$\frac{dz}{dx}(\theta_f)=0\text{, де } \frac{3\pi}{2} \le \theta_f \le 2\pi, \qquad (33)$$

As a result of the transformation (32)-(33) we obtain the equation:

$$\operatorname{ctg}\theta_s=\theta_s+2\pi\frac{b}{a}\text{, де } 0 \le \theta_s \le \frac{\pi}{2}, \qquad (34)$$

$$\operatorname{tg}\theta_f=-\left(\theta_f+2\pi\frac{b}{a}\right),\quad \text{де } \frac{3\pi}{2} \le \theta_f \le 2\pi, \qquad (35)$$

So, for example, at a=0,8 m, b=0, h=0,01 m solving the equations (34)-(35) in the Mathematica software package we receive the following form of an elastic rack (fig. 2):

$$\begin{cases} f_1(\theta)=\frac{0{,}8\theta}{2\pi}, \\ f_2(\theta)=\frac{0{,}8\theta}{2\pi}+0{,}01, \\ 0{,}8482 \le \theta \le 4{,}9126. \end{cases} \qquad (36)$$

Taking the elastic properties of the material of the elastic rack (steel 60C2A: modulus of elasticity E=212000 MPa, Poisson's ratio ν=0.28) and the parameters of the geometric figure a=0.8 m, b=0 m, h=0.01 m are calculated in the software complex Mathematica equations (19)-(29) under the action of different forces F_{ex} and F_{ez}.

In addition, to visualize the process of deformation of the elastic rack, we perform parallel modeling in the software package SolidWorks using Simulation. The maximum load was chosen taking into account the specific resistance of the soil, the depth and width of the working body on the considered elastic rack and taken for 2000 H. The interval of load change – 250 H. The scheme of placement of forces and fastening of an elastic rack is presented in fig. 3.

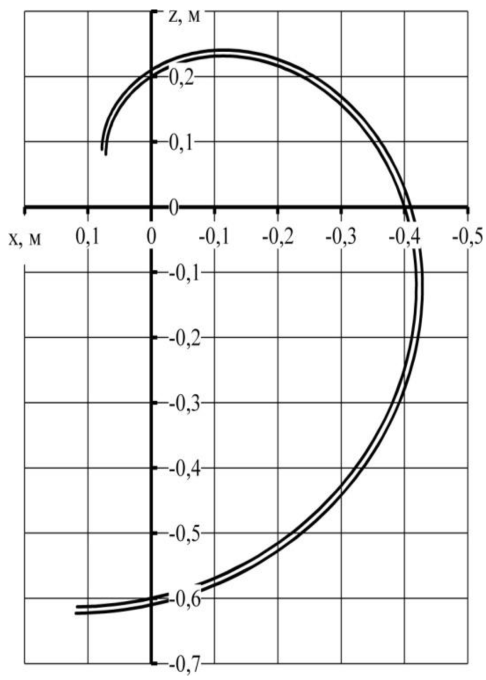

Figure 2. The shape of the elastic rack (36).

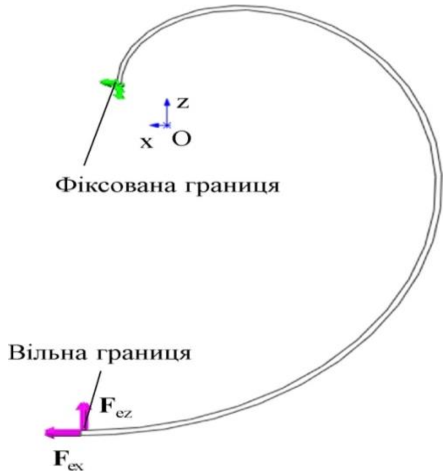

Figure 3. The scheme of forces and fastening of a rack.

The results of the calculation of the deformation of the elastic strut in the software package Mathematica and an example of visualization in the software package SolidWorks are presented in Fig. 4.

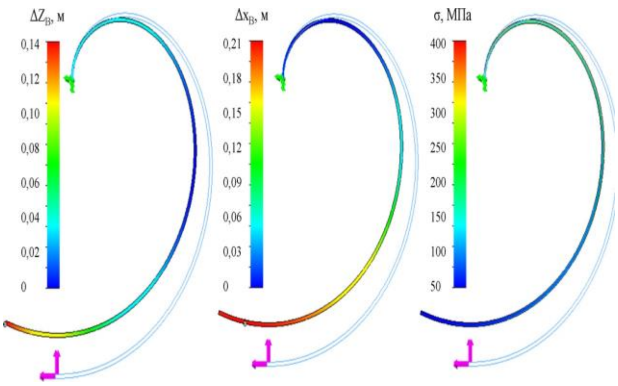

Figure 4. Distribution of stress σ and absolute displacements Δx_B, Δz_B at deformation of an elastic rack with parameters of a geometrical form a = 0.8 m, b = 0 m, h = 0.01 m at F_{ex} = 500 H, F_{ez} = 250 H.

We approximate the obtained data in the form of equations for stiffness coefficients k_x and k_z, length l and angle φ of the equivalent pendulum of an elastic rack with parameters of geometric shape a=0.8 m, b=0 m, h=0.01 m from the values of F_{ex} and F_{ez} forces:

$$k_x = 3457{,}45 + 5{,}16807\, F_{ex} - 0{,}000617126\, F_{ex}^2 - 3{,}32741\, F_{ez} + 0{,}000073511\, F_{ex} F_{ez} + 0{,}000723372\, F_{ez}^2 \text{ (рис. 5)}, \quad (37)$$

$$k_z = 5525{,}65 - 4{,}40952\, F_{ex} + 0{,}000854536\, F_{ex}^2 + 3{,}92764\, F_{ez} + 0{,}00114218\, F_{ex} F_{ez} - 0{,}00134798\, F_{ez}^2 \text{ (рис. 6)}, \quad (38)$$

$$l = 0{,}681292 + 0{,}00011208\, F_{ex} - 1{,}4104\cdot 10^{-8}\, F_{ex}^2 + 0{,}000152196\, F_{ez} - 3{,}16275\cdot 10^{-8}\, F_{ex} F_{ez} + 3{,}4585\cdot 10^{-11}\, F_{ez}^2 \text{ (рис. 7)}, \quad (39)$$

$$\varphi = 1{,}47517 - 0{,}000129246\, F_{ex} + 1{,}85543\cdot 10^{-8}\, F_{ex}^2 - 0{,}000115739\, F_{ez} + 3{,}89549\cdot 10^{-8}\, F_{ex} F_{ez} + 1{,}71867\cdot 10^{-8}\, F_{ez}^2 \text{ (рис. 8)}. \quad (40)$$

The correlation coefficient for equations (37)-(40) is R = 0.99.

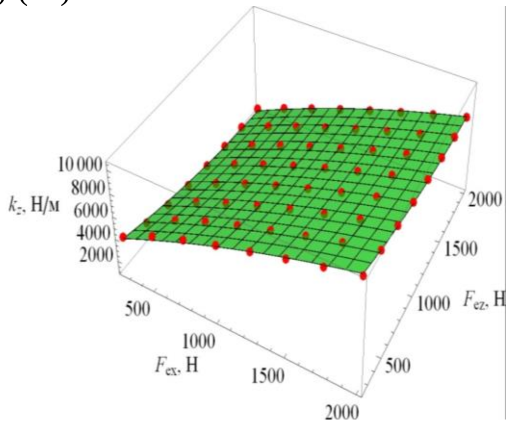

Figure 5. Dependence of the stiffness coefficient k_x at deformation of an elastic rack from parameters of a geometrical form a = 0.8 m, b = 0 m, h = 0.01 m from the forces F_{ex} and F_{ez}

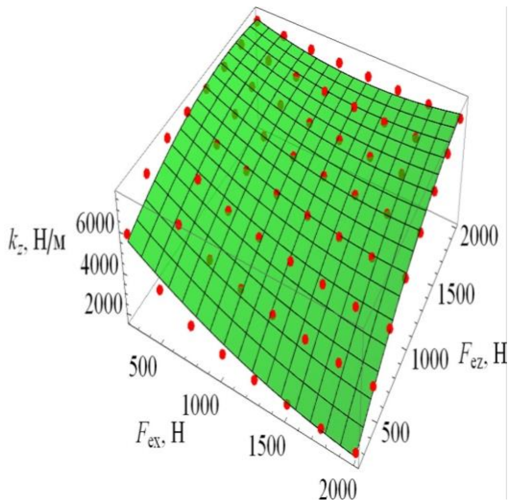

Figure 6. Dependence of the stiffness coefficient k_z at deformation of an elastic rack from parameters of a geometrical form a = 0.8

m, b = 0 m, h = 0.01 m from the forces F_{ex} and F_{ez}

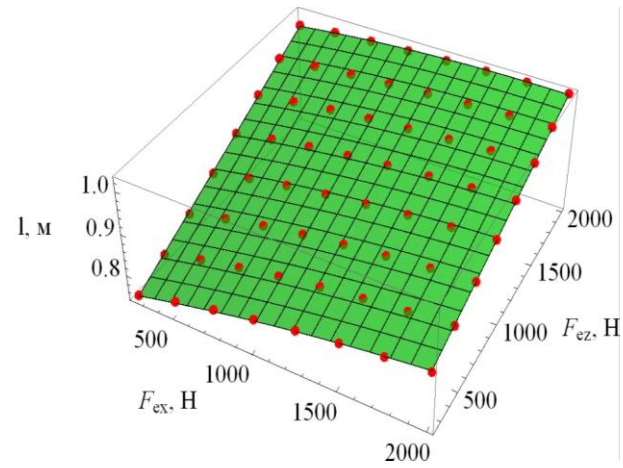

Figure 7. Dependence of the length l of the equivalent pendulum of an elastic rack on the parameters of the geometric shape a = 0.8 m, b = 0 m, h = 0.01 m during its deformation from the forces F_{ex} and F_{ez}

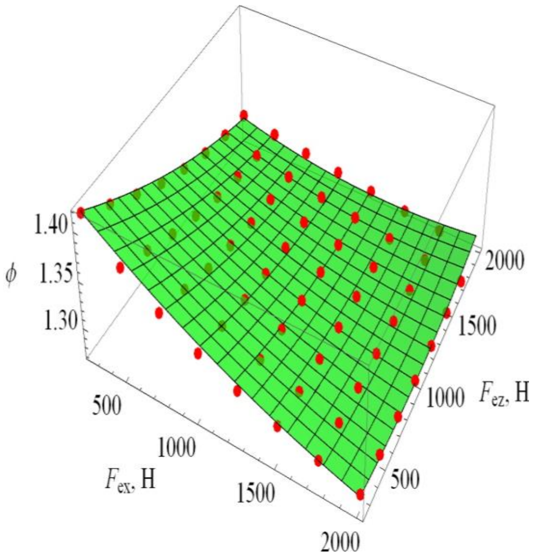

Figure 8. Dependence of the angle of rotation φ of the equivalent pendulum elastic rack with the parameters of the geometric shape a = 0.8 m, b = 0 m, h = 0.01 m during its deformation from the forces F_{ex} and F_{ez}

Thus, based on the results of theoretical studies of the dynamic model of the deformation process of an elastic rack of arbitrary shape of a disk tillage tool, a system of differential equations in general is obtained and the corresponding program code in Mathematica software package is developed. The considered examples for the chosen constructive parameters of an elastic rack of a disc cultivator confirm adequacy of the received theoretical dependences and expediency of their application in an applied aspect at creation of disk working bodies.

CONCLUSIONS

1. As a result of analytical studies of the dynamic model of the process of deformation of the elastic strut of a disc of any shape, a system of differential equations in general is developed and the corresponding program code in Mathematica software package is developed, which allows to determine voltages, relative and absolute deformations.

2. Taking the shape of the elastic rack of the disk for the Archimedean spiral, i.e. the functions of its boundaries are given in polar coordinates, $f_1(\theta)=\frac{a\theta}{2\pi}+b$, $f_2(\theta)=\frac{a\theta}{2\pi}+b+h$, де $\theta_s \le \theta \le \theta_f$, where, from the parameters of the geometric shape a (spiral step), b (spiral displacement along the radial coordinate), h (elastic rack thickness), its equivalent physical-mathematical model in the form of a rigid mathematical pendulum of length l, to the load of which are attached two springs along the axes Ox and Oz with stiffness coefficients k_x and k_z, respectively, which deflect it by an angle φ.

3. The dependences of the stiffness coefficients k_x and k_z, the length l and the angle φ of the equivalent physical and mathematical model of the elastic rack of the disc cultivator with the parameters of the geometric shape a=0.8 m, b=0 m, h=0.01 m from the values of F_{ex} and F_{ez} acting on the free end of the rack along the axes Ox and Oz.

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